Factorization Inevitability in Recursive DAG Admissibility:
Derivation and Current Classification

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Abstract

We establish that admissibility operators over recursive substrate dynamics necessarily factorize into independent structural constraint channels. Using a witness-separability framework, we prove that gate sets satisfying operational independence criteria induce orthogonal failure modes that cannot be reduced to monolithic constraints. We then prove a correspondence theorem: any genuinely independent new gate discovered in extended domains must correspond either to refinement of existing DAG-level invariant classes or to extension of the substrate model itself. This establishes factorization as an inevitable organizing principle rather than an empirical accident, while maintaining honest scoping about current channel classification. Within the characterized domain (Tier-1 recursive operators, XLVII-L DAG model), we identify four independent projection channels validated through Chamber LIII completeness testing. The framework provides clear criteria for when additional gates indicate model refinement versus principle falsification.

1 Introduction

Structural theories of substrate emergence face a fundamental challenge: distinguishing necessary architecture from contingent enumeration. A framework that identifies four structural gates might reflect:

- **Empirical accident**: Four constraints happened to be discovered through chamber validation
- **Domain artifact**: Current testing domain contains four constraints, others exist elsewhere
- **Structural necessity**: Four channels are inevitable projections of deeper invariants

Recent work established two critical results:

1. **Cross-Axis Projection** [2]: The Phase P_3 gate set $\{G_1, G_2, G_3, G_4\}$ acts as a measurable selection operator, achieving $\sim 72\%$ contraction of mechanism-class space
2. **Chamber LIII** [1]: Adversarial relaxation testing across 56,877 mechanisms reveals no hidden mechanism classes—all residual structure collapses into a single unified basin at the G3 bifurcation boundary

These results demonstrate selective power and local completeness, but leave open the central question:

Why must admissibility factorize into independent channels at all?

This paper addresses that question through vertical compression rather than horizontal extension. We do not attempt to prove that exactly four gates are globally necessary. Instead, we establish:

- Factorization of admissibility is inevitable given recursive DAG substrate structure
- Independent gates correspond to DAG-level invariant classes via projection
- Current four-gate classification reflects invariant structure within characterized domain
- Additional gates discovered in extended domains indicate model refinement, not framework collapse

1.1 Relation to Prior Work

Phase $\mathbf{P}_3$ factorization [3] decomposed admissibility empirically into four structural gates. This paper asks why such decomposition must occur.

Cross-Axis Projection proved contraction—that the gate set eliminates mechanism families. This paper addresses the complementary question of why those gates form independent channels.

Chamber LIII established local completeness—that gate relaxation reveals no new mechanism classes within the tested domain. This paper provides the theoretical framework for understanding when additional gates would be discovered and what they would signify.

1.2 Architectural Claim

The core structural claim is:

> **Factorization Principle**: Admissibility operators over recursive DAG dynamics necessarily decompose into orthogonal constraint channels corresponding to distinct DAG-level invariant classes.

This is *not* a claim that exactly four channels exist universally. It is a claim that:

1. Factorization is necessary (not contingent)
2. Channel count reflects DAG invariant dimensionality
3. Current classification (four channels) is well-founded within tested domain
4. Extensions are model refinements, not principle violations

2 Witness-Separability Framework

We formalize channel independence through an operational criterion that can be tested in chambers.

Definition 1 (Gate set and passing region)**.** *Let $\mathcal{M}$ be a mechanism space in a fixed domain (encoding, generator families, protocol). A* gate *is a predicate $G : \mathcal{M} \to \{0, 1\}$. For a finite gate set $\mathcal{G} = \{G_1, \dots, G_k\}$ define the* passing region

$$\mathcal{M}_{\text{pass}}(\mathcal{G}) \;=\; \{m \in \mathcal{M} :\; G_i(m) = 1 \;\forall i\}.$$

*For each i, define the i-*th residual slice

$$\mathcal{F}_i(\mathcal{G}) \;=\; \{m \in \mathcal{M} :\; G_i(m) = 0, \quad G_j(m) = 1 \;\forall j \neq i\}.$$

The residual slices partition the failure space by *first failing gate*. If gates were truly independent, each slice should contain mechanisms whose viability depends specifically on relaxing that gate.

Definition 2 (Witness-separability (independence notion)). *A gate set $\mathcal{G} = \{G_1, \dots, G_k\}$ is* witness-separable *(at protocol $\mathcal{P}$) if for every $i \in \{1, \dots, k\}$ there exists a mechanism $w_i \in \mathcal{M}$ and preregistered $\varepsilon_i > 0$ such that:*

$$w_i \in \mathcal{F}_i(\mathcal{G}) \quad \text{and} \quad w_i \in \mathcal{M}_{\text{viable}}\big(\mathcal{G}[i \leftarrow \varepsilon_i]\big),$$

while for all $j \neq i$ and all preregistered $\varepsilon \leq \varepsilon_{\max}$:

$$w_i \notin \mathcal{M}_{\text{viable}}\big(\mathcal{G}[j \leftarrow \varepsilon]\big).$$

Operationally: each gate has a single-gate witness *that violates it while passing all other gates, and becomes viable when only that specific gate is relaxed.*

This operational definition connects to chamber validation: witness-separability can be tested by searching for mechanisms in each residual slice that become viable under single-gate relaxation.

Definition 3 (Relaxation operator (single-gate)). *Fix a relaxation family for each gate: for $\varepsilon \geq 0$, G_i^{ε} is a relaxed version of G_i with $G_i^0 = G_i$ and monotonicity $G_i^{\varepsilon_1}(m) = 1 \Rightarrow G_i^{\varepsilon_2}(m) = 1$ for $\varepsilon_2 \geq \varepsilon_1$. For a gate set $\mathcal{G}$ define the* single-gate relaxation *of G_i at tolerance ε:*

$$\mathcal{G}[i \leftarrow \varepsilon] \;=\; \{G_1, \dots, G_{i-1}, G_i^{\varepsilon}, G_{i+1}, \dots, G_k\}.$$

Definition 4 (Witness test (protocol aligned)). *Fix a preregistered protocol $\mathcal{P}$ (replicates, seeds, observables, regime classification). A mechanism m is* viable *if it passes $\mathcal{P}$; write $m \in \mathcal{M}_{\text{viable}}$. A* witness for gate G_i *is a mechanism w_i such that:*

$$w_i \in \mathcal{F}_i(\mathcal{G}) \quad \text{and} \quad w_i \in \mathcal{M}_{\text{viable}}\big(\mathcal{G}[i \leftarrow \varepsilon]\big) \text{ for some preregistered } \varepsilon > 0.$$

This formalizes what Chamber LIII tested empirically: whether residual mechanisms become viable when specific gates are relaxed.

The witness-separability criterion is operational—it can be tested through chamber experiments. Proposition 1 (below) establishes that if such witness structure is observed empirically, then factorization becomes necessary for descriptive adequacy, not merely convenient.

Definition 5 (Channel decomposition induced by witnesses). *Let $\mathcal{G}$ be a gate set. A* channel decomposition *of admissibility in the domain is the assignment of failure explanations by* first failing gate*:*

$$\text{fail}_{\mathcal{G}}(m) \;=\; \min\{i : \; G_i(m) = 0\},$$

with the convention that $\text{fail}_{\mathcal{G}}(m) = \perp$ *if $m \in \mathcal{M}_{\text{pass}}(\mathcal{G})$. The induced* failure channels *are the slices $\mathcal{F}_i(\mathcal{G})$.*

3 Factorization Inevitability

We now prove that witness-separability implies necessary factorization.

Theorem 1 (Factorization Inevitability via Witness-Separability). *Fix a domain $(\mathcal{M}, \mathcal{P})$ and a structural gate set $\mathcal{G} = \{G_1, \dots, G_k\}$ with preregistered relaxation families G_i^{ε}.*

If $\mathcal{G}$ is witness-separable at $\mathcal{P}$, then admissibility in the tested domain irreducibly factorizes into k independent constraint channels in the following sense:

1. *(**Irreducibility**) For each i, gate G_i is not implied by the conjunction of the remaining gates:*

$$\exists\, w_i \in \mathcal{M} \text{ s.t. } \bigwedge_{j \neq i} G_j(w_i) = 1 \text{ and } G_i(w_i) = 0.$$

2. *(**Channel-specific unlock**) Each gate i has witness mechanisms whose viability is unlocked by relaxing that channel alone, not by relaxing any other single channel.*

3. *(**Orthogonal failure modes**) The failure channels $\{\mathcal{F}_i(\mathcal{G})\}_{i=1}^k$ are pairwise distinguishable by witnesses: for $i \neq j$, witness w_i lies in $\mathcal{F}_i(\mathcal{G})$ but not in $\mathcal{F}_j(\mathcal{G})$.*

Consequently, no channel is redundant, and each supports a distinct witness class testable through chamber relaxation protocols.

Proof. Item (1) follows immediately from the existence of witnesses $w_i \in \mathcal{F}_i(\mathcal{G})$.

Item (2) is the witness-separability definition: each w_i becomes viable under $\mathcal{G}[i \leftarrow \varepsilon_i]$ but not under any other single-gate relaxation $\mathcal{G}[j \leftarrow \varepsilon]$ for $j \neq i$. This establishes that gate G_i captures variance in viability not accountable to other gates—the operational meaning of channel independence.

Item (3) holds by construction: w_i violates G_i and satisfies all $G_{j \neq i}$, hence $w_i \in \mathcal{F}_i(\mathcal{G})$ and $w_i \notin \mathcal{F}_j(\mathcal{G})$ for $j \neq i$.

Together, these establish that witness-separability implies operational factorization: each gate addresses an independent dimension of admissibility constraint, verifiable through single-gate relaxation experiments. □

Corollary 1 (Single-predicate relaxation cannot reproduce channel structure). *If $\mathcal{G}$ is witness-separable, then no single unified constraint $H : \mathcal{M} \to \{0, 1\}$ with a single relaxation family H^ε can reproduce the channel-specific unlock behavior of $\mathcal{G}$.*

Formally: there exists no relaxation family $\{H^\varepsilon\}_{\varepsilon \geq 0}$ such that for all mechanisms m and all i:

$$m \in \mathcal{M}_{\text{viable}}\big(\mathcal{G}[i \leftarrow \varepsilon]\big) \quad \Longleftrightarrow \quad m \in \mathcal{M}_{\text{viable}}\big(\{H^{\varepsilon'}\}\big) \text{ for some } \varepsilon'.$$

That is: a monolithic pass/fail constraint cannot capture which specific channel *a mechanism violates, only whether it passes or fails globally. Channel information is structurally irreducible.*

Proof. The witnesses $\{w_i\}$ demonstrate that viability under relaxation depends on *which* gate is relaxed, not merely whether *some* constraint is relaxed. Witness w_i becomes viable under $\mathcal{G}[i \leftarrow \varepsilon_i]$ but not under $\mathcal{G}[j \leftarrow \varepsilon]$ for any $j \neq i$. A single relaxation family for H cannot distinguish these cases—it can only vary a single parameter ε, not encode channel identity. □

Proposition 1 (Descriptive adequacy requires channel separation). *Fix a domain $(\mathcal{M}, \mathcal{P})$ where chamber experiments reveal gate-specific unlock behavior: for each gate G_i in a candidate gate set $\mathcal{G}$, there exist mechanisms that become viable when G_i is relaxed but not when any other single gate is relaxed.*

Then any admissibility description adequate to reproduce chamber outcomes must include at least k structurally separable channels.

Formally: if witness-separable unlock behavior is empirically observed, then factorization into at least k channels is necessary for descriptive adequacy, not merely convenient.

Proof. Suppose an alternative description uses fewer than k channels. Then by pigeonhole principle, at least two witnesses w_i and w_j (with $i \neq j$) must map to the same channel in the alternative description. But w_i is unlocked by relaxing G_i alone, while w_j is unlocked by relaxing G_j alone. If they share a channel, relaxing that channel must unlock both—but this contradicts the observed single-gate unlock behavior where relaxing G_i does not unlock w_j.

Therefore, any description adequate to chamber data must preserve at least k separable channels. □

4 DAG-Invariant Correspondence

We now establish that factorization channels correspond to DAG-level invariant classes.

Definition 6 (Independent extension gate)**.** *Let $\mathcal{G} = \{G_1, \dots, G_k\}$ be witness-separable. A new gate $H : \mathcal{M} \to \{0, 1\}$ is an* independent extension *of $\mathcal{G}$ if the augmented set $\mathcal{G}' = \mathcal{G} \cup \{H\}$ is witness-separable and, in addition, H has a witness w_H such that:*

$$H(w_H) = 0, \quad G_i(w_H) = 1 \ \forall i, \quad \textit{and} \quad w_H \in \mathcal{M}_{\text{viable}}\big((\mathcal{G} \cup \{H^{\varepsilon}\})\big) \ \textit{for some } \varepsilon > 0,$$

while w_H is not made viable by relaxing any single G_i alone within the preregistered relaxations.

This definition captures what it means for a new gate to be *genuinely* independent: it must have witnesses that cannot be explained by existing gates.

Definition 7 (DAG-invariant model class and projection map)**.** *Let D be a class of admissible DAG-structured substrate models. Let $\pi : \mathsf{D} \to \mathcal{M}$ be a fixed projection/realization map into mechanism space. Let $\mathsf{Inv}(\mathsf{D})$ denote the set of invariant classes over D under the equivalence relation $\equiv$ induced by D (e.g. path geometry, node distinguishability, branch structure, causal ordering, etc.). A* projection channel *is a map*

$$\Psi : \mathsf{Inv}(\mathsf{D}) \to \{\textit{gates on } \mathcal{M}\}$$

such that gates are invariant-induced predicates on $\pi(\mathsf{D}) \subseteq \mathcal{M}$.

The key idea is that mechanism-level gates arise as projections of substrate-level invariants. The correspondence theorem formalizes this.

Theorem 2 (Correspondence: Independent New Gate Implies Invariant Refinement)**.** *Fix a DAG model class D, projection $\pi : \mathsf{D} \to \mathcal{M}$, and protocol $\mathcal{P}$. Assume the current gate set $\mathcal{G} = \{G_1, \dots, G_k\}$ is witness-separable on $\pi(\mathsf{D})$ and is realized as a set of projection channels:*

$$\exists \ I_1, \dots, I_k \in \mathsf{Inv}(\mathsf{D}) \ \textit{s.t.} \ G_i = \Psi(I_i) \ \textit{on} \ \pi(\mathsf{D}).$$

Let H be an independent extension gate of $\mathcal{G}$ witnessed on the same domain (with the same preregistered protocol and witness rules).

Then at least one of the following must hold:

1. ***(Invariant refinement)*** *There exists an invariant class $I \in \mathsf{Inv}(\mathsf{D})$ whose current treatment in D is not separated by $\{I_1, \dots, I_k\}$, and a refinement $I \rightsquigarrow (I^{(a)}, I^{(b)})$ such that, on $\pi(\mathsf{D})$,*

$$H = \Psi(I^{(a)}) \quad \textit{for some refined invariant } I^{(a)}.$$

Interpretation: H corresponds to a DAG-level invariant channel that was implicit but not formally separated in the current invariant partition.

2. ***(Model extension)*** *The witness behavior defining H cannot be represented as the projection of any invariant class in* $\mathsf{Inv}(\mathsf{D})$ *under the current* π*. In this case the DAG model class must be extended to* $\mathsf{D}' \supset \mathsf{D}$ *(e.g. by adding memory, feedback, non-Markov structure, higher-tier operator structure), so that a new invariant class* $I_H \in \mathsf{Inv}(\mathsf{D}') \setminus \mathsf{Inv}(\mathsf{D})$ *satisfies*

$$H = \Psi'(I_H) \quad \text{on } \pi'(\mathsf{D}').$$

In either case, the correct interpretation of a genuinely independent new gate is refinement or extension of the invariant model*, not falsification of factorization as an organizing principle.*

Proof sketch. Because H is an independent extension, there exists a witness w_H that is not unlocked by relaxing any single existing G_i but is unlocked by relaxing H itself. Therefore H captures variance in viability not accounted for by the existing witness-separable channels $\{G_i\}$.

If H were representable as a Boolean or algebraic composition of $\{G_i\}$ on $\pi(\mathsf{D})$, then no such witness could exist (it would be unlocked whenever the composing gates are unlocked), contradicting independence. Hence H is not reducible to the current channel set on the domain.

If the current invariant partition $\{I_1, \ldots, I_k\}$ is complete for $\mathsf{Inv}(\mathsf{D})$, this contradiction can only be resolved by:

- **Case 1**: Refining the invariant partition to separate previously implicit structure
- **Case 2**: Extending the model class so that the projected invariant set grows

In both cases, the discovery of H indicates that the DAG model D was incompletely specified for the extended domain, not that factorization fails as an organizing principle. □

Corollary 2 (Robustness of factorization principle)**.** *The factorization principle (Theorem 1) survives discovery of new independent gates. Such discoveries refine our understanding of DAG-level structure rather than invalidating the principle that admissibility must factorize.*

5 Current Classification: Four Channels in Characterized Domain

Within the domain characterized by:

- Tier-1 recursive operator pairs (V3×V4, V3×V5, V4×V5)
- DAG model from Chambers XLVII-L
- 4D parameter space (γ_0, β, d, $\kappa_{\max}$)
- Preregistered validation protocol $\mathcal{P}$ from Chamber LIII

we identify four independent projection channels:

5.1 The Phase P_3 Gate Set

1. **G1 (Geometric Curvature)**: Curvature must be computed via turning-angle method
    - DAG invariant: Path geometry
    - Projection: Recursive field must admit curvature-responsive dynamics
2. **G2 (Baseline Separability)**: $\Delta\kappa > 0.05$ required for emergence

- DAG invariant: Node distinguishability
- Projection: Mechanism variants must produce distinguishable state trajectories

3. **G3 (Bifurcation Capability)**: Mechanism must support critical thresholds
   - DAG invariant: Branch structure
   - Projection: Differentiation laws must admit phase transitions

4. **G4 (Locality Consistency)**: No action-at-a-distance coupling
   - DAG invariant: Causal ordering
   - Projection: Recursive updates respect local neighborhood structure

5.2 Validation Evidence

Witness-separability: Empirical completeness testing through adversarial relaxation across 56,877 mechanisms demonstrates single-gate witness behavior [1]. Each gate exhibits characteristic failure modes that become viable under targeted relaxation.

Orthogonality: Contraction analysis shows the gates eliminate distinct mechanism families [2]. The bifurcation gate (G3) concentrates 73-89% of residuals in stress profiles, indicating it captures the critical transition dimension.

Local completeness: Basin unification analysis ($\delta_{\max} = 0.239 < 0.5$) establishes that no fifth independent channel exists within the tested domain [1]. All persistent residual structure collapses into the G3 transition region.

5.3 Honest Scoping

This four-channel classification is *well-founded within the characterized domain* but does not claim global necessity. Extension to:

- Tier-2 operator pairs (V2, V6, V7)
- Higher-dimensional parameter spaces
- Non-recursive interaction laws
- Topological feedback mechanisms

may reveal additional channels. Per Theorem 2, such discoveries would indicate invariant refinement or model extension, not principle falsification.

6 Implications for Structural Theory

6.1 Compression vs. Enumeration

The factorization theorems establish a crucial distinction:

- **Before**: "We found four gates through chamber validation"
- **After**: "Admissibility must factorize; current domain reveals four channels"

This is the difference between:

- **Taxonomy**: Cataloging discovered structures
- **Theory**: Deriving necessity from principles

6.2 Durability Under Refinement

The correspondence theorem (Theorem 2) provides robustness:

> Discovery of a fifth independent gate H does not collapse the framework. It indicates either:
>
> 1. DAG invariant refinement (implicit structure now separated)
> 2. Model extension (new substrate capabilities added)

This makes the theory *refinable* rather than *brittle*.

6.3 Connection to Empirical Validation

The basin unification result from local completeness testing gains theoretical grounding:

- **Empirical finding**: Single residual basin at G3 boundary [1]
- **Theoretical interpretation**: No independent fifth channel exists within current invariant partition

The 73-89% concentration at G3 reflects that bifurcation capability captures the critical dimension where mechanism-space transitions occur.

6.4 Relation to Contraction

Cross-Axis Projection's 72% contraction result is now understood as:

> The four projection channels jointly impose restrictive constraints. Contraction magnitude reflects:
>
> 1. Number of independent channels ($k = 4$)
> 2. Restrictiveness of each DAG invariant
> 3. Interaction structure between channels

Future work may bound contraction ratios from DAG-invariant dimensionality.

7 Future Directions

7.1 Near-Term: Cross-Chamber Validation

Apply the witness-separability protocol across the computational laboratory suite (LII, XLVII, XLVIII, L):

- Test whether four-channel structure replicates across different mechanism domains
- Validate orthogonality of failure modes in diverse contexts
- Measure single-gate relaxation witness behavior systematically

7.2 Theoretical: DAG Invariant Classification

Formalize the invariant classes $\mathsf{Inv}(\mathsf{D})$ categorically:

- Prove four classes exhaust current DAG model
- Or: Identify implicit fifth class requiring separation
- Establish completeness criteria for invariant partitions

7.3 Extension: Tier-2 and Feedback

Test factorization in extended domains:

- Tier-2 operator pairs (V2, V6, V7)
- Non-Markovian feedback mechanisms
- Topological constraints

Per Theorem 2, any new independent gates discovered will refine the DAG model, not falsify the factorization principle.

7.4 Empirical: Projection to Physics

After vertical consolidation, test whether structural factorization has empirical consequences:

- Do bifurcation boundaries project to observable phase transitions?
- Does contraction predict structural bottlenecks in physical law detection?
- Can four-channel structure be tested in spectroscopy or clock precision data?

8 Conclusion

We have established that admissibility over recursive DAG substrate dynamics necessarily factorizes into independent structural constraint channels. This factorization is not an empirical accident but an inevitable consequence of projection from DAG-level invariants to mechanism-class structure.

The witness-separability framework provides operational criteria for testing channel independence in chambers. The correspondence theorem establishes that new independent gates discovered in extended domains indicate DAG model refinement rather than principle falsification.

Within the characterized domain (Tier-1 recursive operators, XLVII-L DAG model, Chamber LIII validation), we classify four independent channels: geometric curvature, baseline separability, bifurcation capability, and locality consistency. Chamber LIII's basin unification result validates that no fifth channel exists within this domain.

This work completes a vertical compression arc:

1. **Phase $\mathbf{P}_3$**: Empirical factorization into four gates
2. **Cross-Axis Projection**: Contraction measurement (72%)
3. **Chamber LIII**: Local completeness validation

4. **This work**: Factorization inevitability and correspondence theorems

The progression from empirical discovery to structural necessity transforms UNNS from a research program into a theory of recursive substrate admissibility.

The decisive conceptual shift is from asking:

"Which gates exist?"

to asking:

"Why must admissibility factorize, and what determines the channel count?"

That is the compression moment.

Acknowledgments

This work builds on Chamber LIII v3.1.0 completeness validation and the Cross-Axis Projection contraction analysis. The witness-separability framework was refined through systematic adversarial testing across 56,877 mechanisms.

References

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